p. 1

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1.	$P(a) + P(b) = 1$
	$(P(a) = P(\bar{b}) \quad , \quad P(b) = P(\bar{a}))$
2.	$P(a \cap a)$
	$= P(a^2)$
3.	$P(b \cap a \cap a)$
	$= P(b \times a^2)$
4.	$P(a \cap b \cap a \cap a)$
	$= P(b \times a^3)$

Points in game (N)	I win	
2	a^2	$(a \cap a)$
3	$b \times a^2$	$(b \cap a \cap a)$
4	$b \times a^3$	$(a \cap b \cap a \cap a)$
5	$b^2 \times a^3$	$(b \cap a \cap b \cap a \cap a)$
6	$b^2 \times a^4$	$(a \cap b \cap a \cap b \cap a \cap a)$
7	$b^2 \times a^4$	$(b \cap a \cap b \cap a \cap b \cap a \cap a)$
8	$b^3 \times a^5$	$(a \cap b \cap a \cap b \cap a \cap b \cap a \cap a)$
9	$b^4 \times a^5$	$(b \cap a \cap b \cap a \cap b \cap a \cap b \cap a \cap a)$
10	$b^4 \times a^6$	$(a \cap b \cap a \cap b \cap a \cap b \cap a \cap b \cap a \cap a)$

Points (N)	If I win a	b
2	$\wedge 2$	0
3	$\wedge 2$	$\wedge 1$
4	$\wedge 3$	$\wedge 1$
5	$\wedge 3$	$\wedge 2$
6	$\wedge 4$	$\wedge 2$
7	$\wedge 4$	$\wedge 3$
8	$\wedge 5$	$\wedge 3$
9	$\wedge 5$	$\wedge 4$
10	[illegible]	$\wedge 4$

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5. The pattern for me winning if the game last for an odd number of point(s) and if the game last for an even number of points is different.

6. For Odd number of points in the game =
Probability (that I win) = [a^($\frac{N}{2}$+0.5)] x [b^($\frac{N}{2}$−0.5)]

For even number of points in the game:
Probability (that I win) = [a^($\frac{N}{2}$+1)] x [b^($\frac{N}{2}$−1)]

The mathematical pattern is the probability that I win has to be: a^x times b^y. a indicates I win, b indicates I don't win.

The relationship between x and y is: x+y = total Number of points in game (N)

I can see that x is 1 unit more than y in odd (N) cases, and x is 2 units more than y in even (N) cases.

In odd (N), number of total points, cases, I begin by loosing the first (N) point.

In even (N), number of total points, cases, I begin by winning the first point.

Then the winning / loosing pattern alternates until I win two points consecutively.

P.3

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7\. If $a = 0.6$, probability that I win a game in 5 points or less =

$\bar{a} = 0.4$ [because $a + \bar{a} = 1$, $1 - a = \bar{a}$, $1 - 0.6 = 0.4$]

P(win in 1 point) ∪ P(win in 2 points) ∪ P(win in 3 points) ∪ P(win in 4 points) ∪ P(win in 5 points)

= P(0.6) + P(0.6 × 0.6) + P(0.4 × 0.6 × 0.6) + P(0.6 × 0.4 × 0.6 × 0.6) + P(0.4 × 0.6 × 0.4 × 0.6 × 0.6)

= P(0.6) + P(0.36) + P(0.144) + P(0.0864) + P(0.03456)

= P(0.62496)

0.62495 rounded up to three decimal points is 0.625. Therefore, the probability that I win a game in 5 points or less (5 points or 4 points or 3 points or 2 points or 1 point) is approximately 0.625.

8\. First, I will test and prove the equation for when I win and the game ends with odd number of points.

Formula:

	$P(\text{I win}) = a^\wedge(\frac{N}{2} + 0.5) \times b^\wedge(\frac{N}{2} - 0.5)$	
Eg1	Probability for game with 3 points $(b \cap a \cap a)$ $= b \times a^2$	If there are 3 points at the end $(N = 3)$: $a^\wedge(\frac{3}{2} + 0.5) \times b^\wedge(\frac{3}{2} - 0.5)$ $= a^\wedge(1.5 + 0.5) \times b^\wedge(1.5 - 0.5)$ $= a^\wedge 2 \times b^\wedge 1$ $= a^2 \times b$.
Eg2	Probability for game with 5 points $(b \cap a \cap b \cap a \cap a)$ $= b^2 \times a^3$	If there are 5 points at the end $(N = 5)$: $a^\wedge(\frac{5}{2} + 0.5) \times b^\wedge(\frac{5}{2} - 0.5)$ $= a^\wedge(2.5 + 0.5) \times b^\wedge(2.5 - 0.5)$ $= a^\wedge 3 \times b^\wedge 2$

The difference between the power of a and power of b is always 1 unit with odd number of points. The power of a always have 1 unit more than the power of B. Because I have to loose one point at the beginning and win with two consecutive points [illegible]

Q 8 (continued) Jessi Lui 10Joy

8. Dividing N by two, since N is an integer will result in a decimal of 0.5. To complete the equation and make difference between power of a and b 1 units, add 0.5 units to a (a have 1 unit more than b) and subtract 0.5 units from b.

I divide N by two because the power of a and b always add up to N and is one unit different. I make the powers of a and b one unit different by adding/subtracting 0.5 units. [0.5 − (−0.5) = 1]

Then, I will test and prove the equation for when I win and the game ends with even number of points.

Formula: $P(I\ win) = [a^{\wedge}(\frac{N}{2}+1)] \times [b^{\wedge}(\frac{N}{2}-1)]$

Eg 1	Probability for game with 2 points ($a \cap a$) $= a^2$	If there are 2 points at the end, ($N=2$) = $[a^{\wedge}(\frac{2}{2}+1)] \times b^{\wedge}(\frac{2}{2}-1)]$ $= a^{\wedge}2 \times b^{\wedge}0$ $= a^{\wedge}2$
Eg 2	Probability for game with 4 points ($a \cap b \cap a \cap a$) $= a^3 \times b$	If there are 4 points at the end ($N=4$): $[a^{\wedge}(\frac{4}{2}+1)] \times [b^{\wedge}(\frac{4}{2}-1)]$ $= [a^{\wedge}(2+1)] \times [b^{\wedge}(2-1)]$ $= a^{\wedge}3 \times b^{\wedge}1$ $= a^3 \times b$

The difference between the power of a and power of b is always 2 units. The power of a always have 2 units more than the power of b. Because I have to begin by winning and end by winning twice (2 extra points than b) for the game to end with even number of points.

Dividing N by two will result the same number for a and b because N is the same integer. I need to add 1 and subtract 1

for the difference between the powers of a and b to be
2 units.